Eur. Phys. J. C 24, 639–642 (2002)
Digital Object Identifier (DOI) 10.1007/s10052-002-0976-0

THE EUROPEAN
PHYSICAL JOURNAL C

The cosmological evolution of the nucleon mass and the electroweak coupling constants

X. Calmet[1,⋆], H. Fritzsch[1,2,⋆⋆]

[1] Ludwig-Maximilians, Sektion Physik, Theresienstraße 37, 80333 München, Germany
[2] Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA

Received: 18 April 2002 /
Published online: 14 June 2002 –

Abstract. Starting from astrophysical indications that the fine structure constant might undergo a small cosmological time shift, we discuss the implications of such an effect from the point of view of particle physics. Grand unification implies small time shifts for the nucleon mass, the magnetic moment of the nucleon and the weak coupling constant as well. The relative change of the nucleon mass is about 40 times larger than the relative change of α. Laboratory measurements using very advanced methods in quantum optics might soon reveal small time shifts of the nucleon mass, the magnetic moment of the nucleon and the fine structure constant.

Some recent astrophysical observations suggest that the fine structure constant α might change with cosmological time [1]. If interpreted in the simplest way, the data suggest that α was lower in the past:

$$\Delta\alpha/\alpha = (-0.72 \pm 0.18) \times 10^{-5} \quad (1)$$

for a redshift $z \approx 0.5 \ldots 3.5$ [1].

The idea that certain fundamental constants might not be constant on a cosmological time scale was pioneered by Dirac [2], Milne [3] and P. Jordan [4]. More recently, time variations of fundamental constants were discussed in connection to theories based on extra dimensions [5].

In this paper we shall study consequences of a possible time dependence of the fine structure constant, which are expected within the framework of the Standard Model of the elementary particle interactions and of unified theories beyond the Standard Model.

In the Standard Model, based on the gauge group $SU(3) \times SU(2) \times U(1)$, the fine structure constant α is not a basic parameter of the theory, but is related to the coupling parameters α_i ($\alpha_i = g_i^2/(4\pi)$, where g_i are the coupling constants of the $SU(3)$, $SU(2)$ or $U(1)$ gauge interactions.

If the three gauge coupling constants are extrapolated to high energy, they come together at an energy of about 10^{16} GeV, as expected, if the QCD gauge group and the electroweak gauge groups are subgroups of a simple gauge group, e.g. $SU(5)$ [6] or $SO(10)$ [7]. Thus the scale of the symmetry breaking of the unifying group determines where the three couplings constants converge [8].

If one takes the idea of grand unification seriously, a small shift in the cosmic time evolution of the electromagnetic coupling constant α would require that the unified coupling constant α_{un} undergoes small time changes as well. Otherwise the grand unification of the three gauge forces would work only at a particular time. Thus in case of a time dependence one should expect, that not only the electromagnetic coupling α, but all three gauge couplings g_1, g_2 and g_3 show such a time variation. One might also consider time changes of other basic parameters, e.g. the electron mass, but here we shall concentrate on the gauge couplings.

Of special interest is a time variation of the QCD coupling g_3. Taking into account only the lowest order in $\alpha_s = g_3^2/(4\pi)$, the behavior of the QCD coupling constant is given by:

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln\left(\frac{\Lambda^2}{\mu^2}\right)} \quad (2)$$

(μ: reference scale, $\beta_0 = -11 + \frac{2}{3}n_f, n_f$: number of quark flavors, Λ: QCD scale parameter). According to the experiments one has $\alpha_s(Q^2 = m_Z^2)_{\overline{\mathrm{MS}}} = 0.1185(20)$. A typical value of the scale parameter Λ is [9]

$$\Lambda = 213^{+38}_{-35}\mathrm{MeV}. \quad (3)$$

If α_s is not only a function of the reference scale μ, but also of the cosmological time, the scale parameter Λ is time-dependent as well.

⋆ Partially supported by the Deutsche Forschungsgemeinschaft, DFG-No. FR 412/27-2,
e-mail: calmet@theorie.physik.uni-muenchen.de
⋆⋆ Partially supported by the VW-Stiftung Hannover (I-77495)

One finds:

$$\frac{\dot{\alpha}_s}{\alpha_s} = \frac{2}{\ln\left(\frac{\mu^2}{\Lambda^2}\right)}\left(\frac{\dot{\Lambda}}{\Lambda}\right). \quad (4)$$

We note that in this relation the coefficient β_0 has cancelled out.

The relative changes $\frac{\delta\alpha}{\alpha}$ and $\frac{\delta\Lambda}{\Lambda}$ are related by: $\left(\frac{\delta\Lambda}{\Lambda}\right) = \left(\frac{\delta\alpha_s}{\alpha_s}\right)\ln(\mu/\Lambda)$. Thus a relative time shift of α_s (likewise α_2 and α_1) cannot be uniform, i.e. identical for all reference scales, but changes logarithmically as the scale μ changes. If one would identify a relative shift $(\delta\alpha_s/\alpha_s)$ at very high energies, say close to a scale $\Lambda_G \approx 10^{16}$ GeV, given e.g. in a grand unified theory of the electroweak and strong interactions, the corresponding relative shift of Λ would be larger by a factor $\ln(\mu/\Lambda) \approx 38$.

In QCD the proton mass, as well as all other hadronic masses are proportional to Λ, if the quark masses are set to zero: $M_p = \text{const.}\ \Lambda$. The masses of the light quarks m_u, m_d and m_s are small compared to Λ, however the mass term of the "light" quarks u, d and s contributes to the proton mass. In reality the masses of the light quarks m_u, m_d and m_s are non-zero, but these mass terms contribute only a relatively small amount (typically less than 10%) to the mass of the nucleon or nucleus. Here we shall neglect those contributions. The mass of the nucleon receives also a small contribution from electromagnetism of the order of 1%, which we shall neglect as well.

If the QCD coupling constant α_s or likewise the QCD scale parameter Λ undergoes a small cosmological time shift, the nucleon mass as well as the masses of all atomic nuclei would change in proportion to Λ. Such a change can be observed by considering the mass ratio m_e/m_p. Since a change of Λ would not affect the electron mass, the electron-proton mass ratio would change in cosmological time.

The three coupling constants α_1, α_2 and α_s seem to converge, when extrapolated to very high energies, as expected in grand unified theories. However, in the Standard Model they do not meet at one point, as expected e.g. in the simplest $SU(5)$-theory [6].

In models based on the gauge group $SO(10)$ [7] a convergence of the three coupling constants can be achieved, if intermediate energy scales are considered [10]. In the minimal supersymmetric extension of the Standard Model the three gauge coupling constants do meet at one point [11].

We consider a theory where the physics affecting the unified coupling constant is taking place at a scale above that of the unification. The main assumption is that the physics responsible for a cosmic time evolution of the coupling constants takes place at energies above the unification scale. This allows to use the usual relations from grand unified theories to evolve the unified coupling constant down to low energy. For example, in string theory the coupling constants are expectation values of fields. They might have some cosmological time evolution [12]. But, at energies below the grand unification point, the usual quantum field theory remains valid.

Whatever the correct unification theory might be, one expects in general that a cosmological time shift affects primarily the unified single coupling constant α_{un}, defined e.g. at the point of unification. In order to be specific, we shall consider the supersymmetric $SU(5)$ grand unified theory broken to the gauge group of the minimal supersymmetric extension of the Standard Model (MSSM) to derive consequences for low energy physics. As usual the scale for supersymmetry breaking is assumed to be in the TeV range. However, our main conclusions will not depend significantly on this assumption.

The scale evolution of the coupling constants in the 1-loop approximation is given by the well-known relation

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i^0(\mu^0)} + \frac{1}{2\pi} b_i \ln\left(\frac{\mu^0}{\mu}\right). \quad (5)$$

The parameters b_i are given by $b_i^{SM} = (b_1^{SM}, b_2^{SM}, b_3^{SM}) = (41/10, -19/6, -7)$ below the supersymmetric scale and by $b_i^S = (b_1^S, b_2^S, b_3^S) = (33/5, 1, -3)$ when $\mathcal{N} = 1$ supersymmetry is restored.

Suppose that the coupling constants α_i depend not only on the scale μ, but also on the cosmological time t: $\alpha_i(\mu, t)$. Since the coefficients b_i are time independent, one finds

$$\frac{1}{\alpha_i(\mu)}\frac{\dot{\alpha}_i(\mu)}{\alpha_i(\mu)} = \frac{1}{\alpha_i(\mu')}\frac{\dot{\alpha}_i(\mu')}{\alpha_i(\mu')}, \quad i \in \{1, 2, 3\} \quad (6)$$

i.e. the quantity $\alpha_i^{-1}(\dot{\alpha}_i/\alpha_i)$ is scale independent.

Since we have to evolve the coupling constants down to energies below the supersymmetry breaking scale, we have to take into account the fact that supersymmetry is broken at low energy. We thus have, replacing the thresholds of the supersymmetric particles by a simple step function,

$$\begin{aligned} \alpha_i(\mu)^{-1} = &\left(\frac{1}{\alpha_i^0(\Lambda_G)} + \frac{1}{2\pi} b_i^S \ln\left(\frac{\Lambda_G}{\mu}\right)\right)\theta(\mu - \Lambda_S) \\ &+ \left(\frac{1}{\alpha_i^0(\Lambda_S)} + \frac{1}{2\pi} b_i^{SM} \ln\left(\frac{\Lambda_S}{\mu}\right)\right)\theta(\Lambda_S - \mu). \end{aligned} \quad (7)$$

Here Λ_S is the supersymmetry breaking scale and

$$\frac{1}{\alpha_i^0(\Lambda_S)} = \frac{1}{\alpha_i^0(M_Z)} + \frac{1}{2\pi} b_i^{SM} \ln\left(\frac{M_Z}{\Lambda_S}\right) \quad (8)$$

where M_Z is the Z-boson mass and $\alpha_i^0(M_Z)$ is the value of the coupling constant under consideration measured at M_Z. We use the following definitions for the coupling constants:

$$\begin{aligned} \alpha_1 &= 5/3 g_1^2/(4\pi) = 5\alpha/(3\cos^2(\theta)_{\overline{\mathrm{MS}}}) \\ \alpha_2 &= g_2^2/(4\pi) = \alpha/\sin^2(\theta)_{\overline{\mathrm{MS}}} \\ \alpha_s &= g_3^2/(4\pi). \end{aligned} \quad (9)$$

We suppose that the unified coupling constant α_{un} undergoes a time shift $\alpha_{un}(\Lambda_G) \to \alpha'_{un}(\Lambda_G) : \alpha'_{un} - \alpha_{un} = \delta\alpha_{un}$. According to (6) and to the convergence of the three coupling constants at the unification point $\Lambda_G = 1.5\times10^{16}$ GeV with $\alpha_{un} = 0.03853$, one finds:

$$\frac{1}{\alpha_1(\mu)}\frac{\dot\alpha_1(\mu)}{\alpha_1(\mu)} = \frac{1}{\alpha_2(\mu)}\frac{\dot\alpha_2(\mu)}{\alpha_2(\mu)} = \frac{1}{\alpha_s(\mu)}\frac{\dot\alpha_s(\mu)}{\alpha_s(\mu)}. \quad (10)$$

Furthermore one derives from (9)

$$\frac{1}{\alpha_2(\mu)}\frac{\dot\alpha_2(\mu)}{\alpha_2(\mu)} = \frac{3}{8}\frac{1}{\alpha(\mu)}\frac{\dot\alpha(\mu)}{\alpha(\mu)} = \frac{1}{\alpha_s(\mu)}\frac{\dot\alpha_s(\mu)}{\alpha_s(\mu)}. \quad (11)$$

We note that the electroweak mixing angle θ, i.e. the quantity $\sin^2\theta$, will also be time dependent, but only for $\mu \neq \Lambda_G$. At $\mu = \Lambda_G$ it is given by the symmetry value $\sin^2\theta = 3/8$. The factor 3/8 in (11) arises from the factor $5\alpha/(3\cos^2\theta)$ by taking the time dependence of $\sin^2\theta$ explicitly into account.

Using $\mu = M_Z$ as the scale parameter in (3), we obtain at $\mu = M_Z$, using $\alpha_s(M_Z) = 0.121$ [13]:

$$\frac{\dot\alpha}{\alpha} = \frac{8}{3}\frac{\alpha}{\alpha_s}\frac{\dot\alpha_s(\mu)}{\alpha_s(\mu)} = \frac{8}{3}\frac{\alpha}{\alpha_s}\frac{1}{\ln\left(\frac{\mu}{\Lambda}\right)}\frac{\dot\Lambda}{\Lambda} \approx 0.0285\cdot\frac{\dot\Lambda}{\Lambda}. \quad (12)$$

Using the scale invariance of $\alpha^{-1}\dot\alpha/\alpha$, we obtain

$$\begin{aligned}\frac{\dot\alpha}{\alpha}(\mu=0) &= \frac{\dot\alpha}{\alpha}(\mu=M_Z)\,\frac{\alpha(\mu=0)}{\alpha(M_Z)} \quad (13)\\ &\approx 0.93\cdot\frac{\dot\alpha}{\alpha}(\mu=M_Z).\end{aligned}$$

The result is:

$$\frac{\dot\Lambda}{\Lambda} = R\frac{\dot\alpha}{\alpha}(\mu=0) \quad (14)$$

the coefficient R is calculated to $R = 37.7\pm2.3$. The uncertainty of R is given, according to (12), by the uncertainty of the ratio α/α_s, which is dominated by the uncertainty of α_s.

We should like to emphasize that the relation (14) is independent of the details of the evolution of the coupling constants at very high energies, in particular it is independent of the details of supersymmetry breaking. The Landau pole of (7) for $i=3$ corresponds to

$$\Lambda = \Lambda_S \exp\left(\frac{2\pi}{b_3^{SM}}\frac{1}{\alpha_{un}}\right)\left(\frac{\Lambda_G}{\Lambda_S}\right)^{\left(\frac{b_3^S}{b_3^{SM}}\right)}. \quad (15)$$

We find

$$\frac{\dot\Lambda}{\Lambda} = -\frac{3}{8}\frac{2\pi}{b_3^{SM}}\frac{1}{\alpha}\frac{\dot\alpha}{\alpha}. \quad (16)$$

i.e. there is no dependence on Λ_S. If we calculate $\dot\Lambda/\Lambda$ using the relation above in the case of 6 quark flavors, neglecting the masses of the quarks, we find $R \approx 46$.

This shows that the actual value of R is sensitive to the inclusion of the quark masses and the associated thresholds, just like in the determination of Λ. Furthermore higher order terms in the QCD evolution of α_s will play a role. For this reason the systematic uncertainty in the value of R is certainly larger than the error given above. We estimate:

$$R = 38 \pm 6 \quad (17)$$

taking into account both the experimental error in the determination of $\alpha_s(M_Z)$ and the systematic uncertainties.

The time change of Λ implies a time change of the proton mass and of all nuclear mass scales, as well as of the pion mass, which would change in proportion to $\Lambda^{1/2}$, according to the chiral symmetry realtion $M_\pi^2 = \text{const}.m_q\Lambda$ (m_q: light quark mass average). We obtain

$$\frac{\dot M}{M} = \frac{\dot\Lambda}{\Lambda} = R\frac{\dot\alpha}{\alpha} \approx 38\cdot\frac{\dot\alpha}{\alpha}. \quad (18)$$

Thus the change of the nucleus mass amounts to about 0.3 MeV, if we base our calculations on the time shift of α given in [1]. At a redshift of about one the mass of the nucleon as well as the masses of the nuclei were about 0.3 $^0/_{00}$ smaller than today.

In QCD the magnetic moment of the proton $\mu = g_p\cdot e/2M_p$ is related to the magnetic moments of the constituent quarks. Although it is not possible to calculate the magnetic moment of the proton with high precision, the moment scales in proportion to Λ^{-1} in the chiral limit where the quark masses vanish. Thus, we have

$$\frac{\dot\mu_p}{\mu_p} = -\frac{\dot\Lambda}{\Lambda} = -R\frac{\dot\alpha}{\alpha}. \quad (19)$$

The gyromagnetic ratio g_p will not be time dependent, since the proton mass scales like μ_p^{-1}, however the ratio of the magnetic moments μ_p/μ_e will be time-dependent:

$$\dot{\left(\frac{\mu_p}{\mu_e}\right)}\Big/\left(\frac{\mu_p}{\mu_e}\right) = -\frac{\dot\Lambda}{\Lambda} = -R\frac{\dot\alpha}{\alpha}. \quad (20)$$

The present astrophysical limit on the proton-electron mass ratio $\mu = M_p/m_e$ obtained at a redshift of $z = 2.81$ is [14]

$$-1.7\times10^{-5} < \frac{\Delta\mu}{\mu} < 2\times10^{-4}. \quad (21)$$

Using (18) and (1), one would expect:

$$\frac{\Delta\mu}{\mu} \approx -3\cdot10^{-4} \quad (22)$$

a result, which violates the bound (21), but in view of the large errors on the astrophysical side we do not regard this as a serious disagreement, rather as a sign that astrophysical data might soon clarify whether a time change of the nuclear mass scale following (18) is indeed present.

A clarification of the situation could come from laboratory experiments. Assuming an age of the universe of the order 14 Gyr, the various astrophysical limit can be used to derive relative changes of the various quantities, e.g. $\dot{\alpha}/\alpha$ or $\dot{\Lambda}/\Lambda$, per year, assuming for simplicity a linear time evolution. The constraint on $|\Delta\mu_p/\mu_p|$ given above (21) leads to [14]:

$$\left|\frac{\dot{\mu}_p}{\mu_p}\right| < 1.5 \times 10^{-14}\ \mathrm{yr}^{-1}. \qquad (23)$$

Direct laboratory measurements provide the constraint [15]:

$$\left|\frac{\dot{\alpha}}{\alpha}\right| \leq 3.7 \times 10^{-14}\ \mathrm{yr}^{-1}. \qquad (24)$$

Using advanced methods in quantum optics, it seems possible to improve the present laboratory limits for a time variation of α and of the nucleon mass by several orders of magnitude. A time variation of α could be observed by monitoring the atomic fine structure in a period of several years. Monitoring the rotational and/or vibrational transition frequencies of molecules, e.g. diatomic molecules like H_2 or CO would allow to set stringent limits on a time variation of the nucleon mass.

According to our estimates, the largest effect is expected to be a cosmological time shift of the nucleon mass, observed e.g. by monitoring molecular frequencies. Due to the relation (19) similar effects (same amounts, opposite sign) should be seen in a time shift of μ_p, observed by monitoring hyperfine transitions. These effects should be about 40 times larger than a time shift of α (see (14)), observed e.g. in monitoring fine structure effects. In quantum optics one may achieve a relative accuracy in frequency measurements of the order of $\Delta\omega/\omega \approx 10^{-18}$, which would allow to improve the present limits significantly or observe effects of time variation. We note, however that the present continuously operated atomic frequency standards (H, Cs, Hg^+) are using transitions between ground states hyperfine energy levels, given by the interaction of a nuclear magnetic moment with the magnetic moment of the valence electron [15]. In a relative comparison the time dependence of the nuclear magnetic moments drops out. In order to see an effect, following (19), a comparison with a frequency standard independent of the nuclear magnetic moments is necessary.

It is quite possible that future laboratory experiments find positive effects for time variations of M_p, μ_p and α. If a time variation is observed, the actual amount of time variation, say the value of $\dot{M}/M$, would be an important parameter to connect particle physics quantities with the cosmological evolution.

Finally we should like to mention that the link between the various coupling constants of the Standard Model discussed here implies that nuclear physics scales, including the pion mass, change as well. For this reason the constraints on a time variation of α derived from an analysis of the natural reactor at Oklo (Gabon, Africa) [16] cannot be taken seriously. In fact, it is a bound on the product αM_π under the additional assumption that other nuclear physics and strong interaction parameters do not change. The product αM_π would change, according to the relation (14) as $\dot{\alpha}/\alpha + \dot{\Lambda}/(2\Lambda) \approx 21\dot{\alpha}/\alpha$, since M_π is proportional to $\sqrt{m\Lambda}$ (m: light quark mass). This would lead to a bound about an order of magnitude stronger than the present bound on the time variation of α. However other nuclear physics parameters, change as well. A more detailed analysis of the nuclear physics aspects of a time change of Λ is needed in order to see whether there is a disagreement here.

Furthermore we expect a small cosmological time shift of the $n-p$ mass difference. This would affect the cosmic nucleosynthesis of the light elements. An analysis of nucleosynthesis will be made elsewhere.

Acknowledgements. We are grateful to T. Haensch and to H. Walther for discussions on the potential of the measurements of the time variation of α and of the nucleon mass using quantum optics methods. Furthermore we are indebted to E. Kolb, A. Hebecker, J. March-Russell, P. Minkowski and G. Steigman for discussions and to T. Dent for a useful communication.

References

1. J.K. Webb et al., Phys. Rev. Lett. **87** 091301 (2001) [arXiv:astro-ph/0012539]
2. P.M. Dirac, Nature **192**, 235 (1937)
3. E.A. Milne, Relativity, Gravitation and World Structure, Clarendon press, Oxford, (1935), Proc. Roy. Soc. A, 3, 242 (1937)
4. P. Jordan, Naturwiss., 25, 513 (1937), Z. Physik, 113, 660 (1939)
5. W.J. Marciano, Phys. Rev. Lett. **52**, 489 (1984), G.R. Dvali, M. Zaldarriaga, arXiv:hep-ph/0108217, T. Banks, M. Dine, M.R. Douglas, hep-ph/0112059
6. H. Georgi, S.L. Glashow, Phys. Rev. Lett. **32**, 438 (1974)
7. H. Fritzsch, P. Minkowski, Annals Phys. **93**, 193 (1975), H. Georgi, in Particles and Fields, (AIP, New York, 1975)
8. H. Georgi, H.R. Quinn, S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974)
9. S. Bethke, J. Phys. G **26**, R27 (2000) [arXiv:hep-ex/0004021]
10. D. Chang, R.N. Mohapatra, M.K. Parida, Phys. Rev. D **30**, 1052 (1984), R.N. Mohapatra, M.K. Parida, Phys. Rev. D **47**, 264 (1993) [arXiv:hep-ph/9204234]
11. U. Amaldi, W. de Boer, H. Furstenau, Phys. Lett. B **260**, 447 (1991)
12. M.B. Green, J.H. Schwarz, E. Witten, Superstring Theory, Vol. 1 and 2, T. Damour, A.M. Polyakov, Nucl. Phys. B **423**, 532 (1994) [arXiv:hep-th/9401069]
13. D.E. Groom et al. [Particle Data Group Collaboration], Eur. Phys. J. C **15**, 1 (2000)
14. A.Y. Potekhin, A.V. Ivanchik, D.A. Varshalovich, K.M. Lanzetta, J.A. Baldwin, G.M. Williger, R.F. Carswell, Astrophys. J. **505**, 523 (1998) [arXiv:astro-ph/9804116]
15. J.D. Prestage, L.T. Robert, L. Maleki, Phys. Rev. Lett. **74** 3511 (1995)
16. A.I. Shylakhter, Nature, **264**, 340 (1976); ATOMKI Report A/l, 1983